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The New Criteria for H – Matrices

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Abstract: Research in biology, economics, mathematics and many other disciplines have an important application of nonsingular H – matrix judgment problem, in the H – matrix a subclass of matrix α_1 – strictly diagonally dominant matrix, with one of the important theorem α_1 – strictly diagonally dominant theorem by using the construction certificate is obtained by the method of generalized strictly diagonally dominant matrix (nonsingular H – matrix) new concise and practical criterion.
Key words: generalized strictly diagonally dominant matrix; nonsingular H – matrix; strictly α_1 – diagonally dominant matrix; positive diagonal matrix
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H – 矩阵的新的判定条件

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摘要：研究了在生物学、经济学、计算数学等许多学科中都有重要应用的非奇异 H – 矩阵的判断问题，在 H – 矩阵的一类子矩阵 α_1 – 严格对角占优矩阵下，借助其中的重要定理 α_1 – 严格对角占优定理，并利用构造性证明法得到了广义严格对角占优矩阵（非奇异 H – 矩阵）新的简洁实用的判据，同时数值算例也表明此方法的有效性.
关键词：广义严格对角占优矩阵；非奇异 H – 矩阵；严格 α_1 – 对角占优矩阵；正对角矩阵

1 Introduction

Generalized strictly diagonally dominant matrices (nonsingular H – matrix) play a vital role in computational mathematics, mathematical physics, control theory and other fields (In these years have get some criteria for H – matrix[1-4]), the α_1 – diagonally dominant theory have been proved to be very useful in this problem. In this paper, a new criteria for generalized strictly diagonally dominant matrices is obtained by using the α_1 – diagonally dominant theory. The numerical example verifies the validity of this criteria.

Let $A = (a_{ij}) \in C^{n\times n}$, $R_i(A) = \sum_{j\neq i} |a_{ij}|$, $C_i(A) = \sum_{i\neq j} |a_{ji}|$, $N = \{1,2,\cdots,n\}$. A is called a strictly diagonally dominant matrix, if $|a_{ii}| > R_i(A)$, $\forall i \in N$, denote $A \in D_\circ$. Hypothesis positive diagonal matrix D meet $AD \in D_\circ$, A called the generalized strictly diagonally dominant matrix, mark $A \in D_\circ^*$ (i. e., A is a nonsingular H – matrix).

Definition 1 [1] Let $A = (a_{ij}) \in C^{n\times n}$, if there is $\alpha_1 \in (0,1]$ such that $|a_{ii}| > \alpha R_i + (1-\alpha)C_i$, $\forall i \in N$. So A called a strictly α_1 – diagonally dominant matrix, mark $A \in D_\circ(\alpha_1)$, in which $R_i = R_i(A)$, $C_i = C_i(A)$. Hypothesis positive diagonal matrix D meet the matrix $AD \in D_\circ(\alpha_1)$, A called a generalized strictly α_1 – diagonally dominant matrix, mark $A \in D^*(\alpha_1)$.

Denote $N_1 = \{i \in N: 0 < |a_{ii}| \leq \alpha R_i + (1-\alpha)C_i\}$, $N_2 = \{i \in N: |a_{ii}| > \alpha R_i + (1-\alpha)C_i\}$, And $A(N_1)$ is a sub-matrix of A whose rows and columns are indexed by N_1.

If $N_1 = \varnothing$, A is a strictly α_1 – diagonally dominant matrix, and if $N_2 = \varnothing$, A can not be a generalized strictly α_1 – diagonally dominant matrix, so only the condition that N_1, $N_2 \neq \varnothing$ is considered in this paper.

Lemma 1 [2] $A = (a_{ij}) \in C^{n\times n}$ if and only if the following conditions can be satisfied, so $A \in D_\circ^*$ (nonsingular H – matrix).

1) $A \in D_\circ(\alpha_1)$;

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2) There are two positive diagonal matrices D_1 and D_2 such that $D_1AD_2 \in D_\circ^*$.

We can get the conclusion from Lemma 1 that if $A \in D^*(\alpha_1)$, then A is a generalized H – matrix.

2 Main Results

Theorem 1 If $A \in D_\circ(\alpha_1)$, so $A(N_1) \in D^*(\alpha_1)$.

Proof Assume that $A(N_1) \notin D^*(\alpha_1)$, we employ the contradiction method to prove that this case is impossible.

If the assumption is true, then $\forall D = \mathrm{diag}(d_1, d_2, \cdots, d_n)$ $(d_i > 0)$, there exists $k \in N_1$ such that

$$|a_{kk}d_k| \le \alpha\Big(\sum_{j\in N_1, j\neq k}|a_{kj}d_j|\Big) + (1-\alpha)\Big(\sum_{j\in N_1, j\neq k}|a_{jk}d_k|\Big). \tag{1}$$

Because $A \in D_\circ(\alpha_1)$, so there is a positive diagonal matrix D' , $D' = \mathrm{diag}(d'_1, d'_2, \cdots, d'_n)$ such that $AD' \in D_\circ(\alpha_1)$, namely

$$|a_{kk}d'_k| > \alpha\Big(\sum_{j\neq k}|a_{kj}d'_j|\Big) + (1-\alpha)\sum_{j\neq k}|a_{jk}d'_k| \ge \alpha\Big(\sum_{j\in N_1, j\neq k}|a_{kj}d'_j|\Big) + (1-\alpha)\Big(\sum_{j\in N_1, j\neq k}|a_{jk}d'_k|\Big). \tag{2}$$

(1) is in contradiction with (2), so the assumption is false, i. e. , $A(N_1) \in D^*(\alpha_1)$.

Theorem 2 $A = (a_{ij}) \in C^{n\times n} \in D_\circ^*$ if the following 1), 2) established

1) $A(N_1) \in D_\circ^*$ and exists $d_i > 0$ $(i \in N_1)$, such that

$$|a_{kk}d_k| > \alpha\Big(\sum_{j\in N_1, j\neq k}|a_{kj}d_j|\Big) + (1-\alpha)\Big(\sum_{j\neq k}|a_{jk}d_k|\Big) \quad (\forall k \in N_1); \tag{3}$$

2) There exists a positive number ε , $\forall i \in N_2$.

$$|a_{ii}| > \alpha\Big(\sum_{j\in N_1}|a_{ij}|d_j(\max_{k\in N_1}\{r_k\}+\varepsilon) + \sum_{j\in N_2, j\neq i}|a_{ij}|\Big) + (1-\alpha)\Big(\sum_{j\neq i}|a_{ji}|\Big). \tag{4}$$

where $r_k = \Big(\sum_{j\in N_2}|a_{kj}|\Big)\left[\frac{|a_{kk}d_k| + (\alpha-1)\sum_{j\neq k}|a_{jk}d_k|}{\alpha} - \sum_{j\in N_1, j\neq k}|a_{kj}d_j|\right]^{-1}$, $k \in N_1$.

Specially, if $\frac{|a_{kk}d_k| + (\alpha-1)\sum_{j\neq k}|a_{jk}d_k|}{\alpha} - \sum_{j\in N_1, j\neq k}|a_{kj}d_j| = 0$, the $r_k = 1$.

Proof Construct a positive diagonal matrix $D' = \mathrm{diag}(d'_1, d'_2, \cdots, d'_n)$,

where
$$d'_i = \begin{cases} d_i(\max_{k\in N_1}\{r_k\}+\varepsilon), & i \in N_1; \\ 1, & i \in N_2. \end{cases}$$

Let $B = AD'$, $\forall i \in N_1$,

$$\max_{k\in N_1}\{r_k\}+\varepsilon > r_i = \Big(\sum_{j\in N_2}|a_{ij}|\Big)\left[\frac{|a_{ii}d_i| + (\alpha-1)\sum_{j\neq i}|a_{ji}d_i|}{\alpha} - \sum_{j\in N_1, j\neq i}|a_{ij}|d_j\right]^{-1},$$

So
$$(\max_{k\in N_1}\{r_k\}+\varepsilon)\left(\frac{|a_{ii}d_i| + (\alpha-1)\sum_{j\neq i}|a_{ji}d_i|}{\alpha} - \sum_{j\in N_1, j\neq i}|a_{ij}d_j|\right) > \sum_{j\in N_2}|a_{ij}|,$$

then $(\max_{k\in N_1}\{r_k\}+\varepsilon)|a_{ii}|d_i + (\alpha-1)(\max_{k\in N_1}\{r_k\}+\varepsilon)\Big(\sum_{j\neq i}|a_{ji}|d_i\Big) - \alpha(\max_{k\in N_1}\{r_k\}+\varepsilon)\Big(\sum_{j\in N_1, j\neq i}|a_{ij}|d_j\Big) > \alpha\sum_{j\in N_2}|a_{ij}|$.

i. e. , $(\max_{k\in N_1}\{r_k\}+\varepsilon)|a_{ii}d_i| > \alpha\Big[\sum_{j\in N_2}|a_{ij}| + (\max_{k\in N_1}\{r_k\}+\varepsilon)\sum_{j\in N_1, j\neq i}|a_{ij}|d_j\Big] + (1-\alpha)\Big[(\max_{k\in N_1}\{r_k\}+\varepsilon)\Big(\sum_{j\neq i}|a_{ji}|d_i\Big)\Big]$.

So
$$|b_{ii}| > \alpha\sum_{j\in N, j\neq i}|b_{ij}| + (1-\alpha)\sum_{j\in N, j\neq i}|b_{ji}| \quad (\forall i \in N_1). \tag{5}$$

From (4), $\forall i \in N_2$,

$$\alpha\sum_{j\in N, j\neq i}|b_{ij}| + (1-\alpha)\sum_{j\in N, j\neq i}|b_{ji}| = \alpha\Big(\sum_{j\in N_1}|a_{ij}|d_j(\max_{k\in N_1}\{r_k\}+\varepsilon) + \sum_{j\in N_2, j\neq i}|a_{ij}|\Big) + (1-\alpha)\sum_{j\in N, j\neq i}|a_{ji}| < |a_{ii}| = |b_{ii}|. \tag{6}$$

From (5) and (6) $|b_{ii}| > \alpha\sum_{j\in N, j\neq i}|b_{ij}| + (1-\alpha)\sum_{j\in N, j\neq i}|b_{ji}|$ $(\forall i \in N)$.

So $B \in D_\circ(\alpha_1)$, namely $A \in D_\circ$.

Corollary 1 Let $A = (a_{ij}) \in C^{n\times n}$ if the following conditions can be satisfied, then A is called a nonsingular H – matrix.

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多地点的表现,在生产推广应用过程中针对可能出现的问题采取配套生产措施.这就需要育种主持单位必须制定相应措施,鼓励区试承担单位和执行人,充分挖掘区试数据资源,发现问题、解决问题,不能回避出现的问题.尤其是病害,室内抗性鉴定是有限的,田间多年、多点发生的病害必须得到重视,并有针对性地进行配套生产技术研究.

5)还可以通过反复培训、继续深造、交流学习、激励自学和岗位技能竞赛等措施,为从事此项工作的人员搭建一个利于个人发展成才的平台,促进这支队伍迅速成长,将他们打造成为服务于“两烟”事业各岗位的行家里手,加快推进我省“两烟”生产健康可持续发展.

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1) $A(N_1) \in D^*(\alpha_1)$, and there exists $d_i > 0$ ($\forall i \in N_1$), such that

$$|a_{kk}d_k| > \alpha\Big(\sum_{j\in N_1, j\neq k}|a_{kj}d_j|\Big) + (1-\alpha)\Big(\sum_{j\neq k}|a_{jk}d_k|\Big)\ (\forall k \in N_1);$$

2) $$\max_{k\in N_1}\{r_k\} < \min_{k\in N_2}\left\{\left[\frac{|a_{kk}| + (\alpha-1)\sum_{j\neq k}|a_{jk}|}{\alpha} - \sum_{j\in N_2, j\neq k}|a_{kj}|\right]\Big(\sum_{j\in N_1}|a_{kj}d_j|\Big)^{-1}\right\}.$$

Similarly the proof Theorem 1.

Example 1 Let $A = \begin{pmatrix} 3 & 2 & 2 & 1 \\ 0.5 & 5 & 5 & 1 \\ 0.5 & 1 & 9 & 9 \\ 1 & 1 & 1 & 8.5 \end{pmatrix}$ and $\alpha = 0.5$, then $N_1 = \{1,2,3\}$, $N_2 = \{4\}$.

Let $d_1 = d_2 = d_3 = 1$, $A(N_1)\ \mathrm{diag}\{d_1, d_2, d_3\}$ is a strictly α_1 − diagonally dominant matrix, and let $\varepsilon = 0.3$,

$$|a_{11}d_1| = 3 > 0.5\Big(\sum_{j\in N_1, j\neq 1}|a_{1j}d_j|\Big) + 0.5\Big(\sum_{j\neq 1}|a_{j1}d_1|\Big) = 2.50,$$

$$|a_{22}d_2| = 5 > 0.5\Big(\sum_{j\in N_1, j\neq 2}|a_{2j}d_j|\Big) + 0.5\Big(\sum_{j\neq 2}|a_{j2}d_2|\Big) = 4.25,$$

$$|a_{33}d_3| = 9 > 0.5\Big(\sum_{j\in N_1, j\neq 3}|a_{3j}d_j|\Big) + 0.5\Big(\sum_{j\neq 3}|a_{j3}d_3|\Big) = 4.25,$$

$$|a_{44}| = 8.5 > 0.5\Big(\sum_{j\in N_1}|a_{4j}|d_j(\max_{k\in N_1}\{r_k\} + \varepsilon) + \sum_{j\in N_2, j\neq 4}|a_{4j}|\Big) + 0.5\Big(\sum_{j\neq 4}|a_{j4}|\Big) \approx 7.5382.$$

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